The Ricardian Equivalence under Collateral Constraints♦

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Abstract

This paper investigates the Ricardian Equivalence (RE) under collateralized debt, default, transaction costs and incomplete markets. The public debt is neutral and the RE holds only if the collateral-transfer cost depends linearly on the lump-sum tax and is fully offset. Lenders and borrowers should enter in a voluntary agreement to compensate for any transfer cost under default. However, any perturbation in the assumed affine relation undermines the debt neutrality. It is not the transaction cost *per se* that invalidates the RE, but rather how this cost affects the households' indebtedness and budget constraint. The underlying mechanism is the credit channel of the fiscal policy. Whenever the transfer cost is not fully offset, there is a net tax balance leftover that affects the budget set and real allocations. This is fundamentally different from a liquidity constrained economy because the credit channel of the fiscal policy is binding and uncompensated transaction costs lead to the RE failure.

Keywords

Ricardian equivalence; Collateral constraints; Debt neutrality.

Resumo

Este artigo investiga a Equivalência Ricardiana (ER) sob dívida colateralizada, inadimplência, custos de transação e mercados incompletos. A dívida pública é neutra e a RE é válida somente

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se o custo da transferência de garantias depender linearmente do imposto lump sum e for totalmente compensado. Credores e tomadores devem celebrar um acordo voluntário para compensar quaisquer custos de transferência em caso de descumprimento. Contudo, qualquer perturbação na relação afim assumida compromete a neutralidade da dívida. Não é o custo de transação em si que invalida a RE, mas sim a forma como este custo afeta o endividamento e a restrição orçamentária das famílias. O mecanismo subjacente é o canal de crédito da política fiscal. Sempre que o custo da transferência não é totalmente compensado, há uma sobra de saldo tributário líquido que afeta o conjunto orçamentário e as dotações reais. Isto é fundamentalmente diferente de uma economia com restrições de liquidez porque o canal de crédito da política fiscal é ativo e os custos de transação não compensados levam ao fracasso da ER.

Palavras chaves
Equivalência Ricardiana; Restrições de colateral; Neutralidade da dívida.

Classificação JEL
D50, H3.

1. Introduction

The debate on the effects of the fiscal policy on the real economy has long divided the economic literature and placed renowned researchers, such as Barro (1974) and Buchanan (1976), on opposite sides of the dispute. The theory underlying this issue is the well-known Ricardian Equivalence (RE). Those in favor of the RE argue that the question is not the public debt *per se*, but rather whether economic agents have the financial mechanisms to neutralize the debt, as claimed by Carmichael (1982). However, as shown by Divino and Orrillo (2022), the economic environments under which the RE holds are usually restrictive and not robust in practice due to required assumptions and lack of empirical evidence.

The RE has been investigated under a variety of circumstances, ranging from hypothesis on agents' behavior to specific market characteristics. In this paper, we focus on a specific financial friction represented by collateral constraints, which have gained relevance since the pioneer work by Kiyotaki and Moore (1997) and not yet explored in the RE literature to the best of our knowledge. Therefore, we also shed light on the conditions under which the fiscal policy might affect the economy-wide consumption path. For recent studies on collateralized economies in a macro-finance environment, we refer to Mendoza (2010), Simsek (2013), Gorton and

Ordoñez (2014) and Guerrieri and Iacoviello (2017). Brunnermeier, Eisenbach, and Sannikov (2012) report an excellent survey covering an older period.

Here, we investigate how the presence of collateralized debt might affect the RE and the neutrality of the fiscal policy through the credit channel. Specifically, we consider a situation in which: i) individuals can only issue collateralized debt; ii) in case of default, the lenders recover only a fraction of the collateral realized value; and iii) the transaction cost is a function of the lump-sum tax. Other than this, the model is a standard version of a two-period GEI economy with two states of nature, uncertainty in the second period, and a single secure asset/liability that pays a risk-free interest rate.

The two-period assumption is not restrictive when it comes to analyze RE under economic frictions. Panadés (2001), for instance, addressed the RE under the fiscal constraint of tax evasion in a two-period economy. That assumption would be restrictive only when there is no prior mechanism to avoid Ponzi games. However, it is well-known that Ponzi games are endogenously ruled out by the presence of collateral, as shown by Araujo et al. (2004).

Regarding the fiscal framework, the government might engage in different tax schemes, including distortionary, agent-specific or lump-sum taxes. Our major objective is to address how collateralized debt subject to default might affect the RE rather than how the RE depends on the economy tax structure. Therefore, we assume the simplest possible tax scheme, given by lump-sum taxes as in Hayford (1989). The RE is usually found to hold under this assumption, which does not play any role to our results. We refer to Bassetto and Kocherlakota (2004) for a discussion on the RE under distortionary taxes.

Collateralized lending has been analyzed by many authors, as documented by Geanakoplos and Zame (2014). We rest on this literature to build a two-period model of a collateralized economy that faces uncertainty and the possibility of default in the housing market in the second period. The model combines the basic framework of Hayford (1989), where the RE is assessed in terms of a debt-financed tax cut, with the debt enforcement proposed by Geanakoplos and Zame (2014). However, we depart from these studies in two major dimen- sions. Firstly, unlike Hayford, our debt enforcement mechanism is the seizure of collateral and not the American personal bankruptcy law used by him, meaning that we extend the RE dependence on the compensation scheme. Secondly, unlike Geanakoplos and Zame, where the collateral transfer in case of default is smooth, we consider borrowing markets in which the collateral transfer bears a legal cost that impacts its future face value. Because of this transfer cost, the state of default occurs whenever the net future value of collateral falls below the face value of the debt.[1] This transfer cost is a key element to dictate whether the public debt might be neutralized.

The intuition behind this condition is that lenders and borrowers might enter in a voluntary agreement to pay for any cost of collateral transfer under default. For this, lenders should give up part of the depreciated collateral value, which would be received by the borrowers as payment to execute the collateral transfer. This additional income could po- tentially serve to neutralize any tax cut enacted by the government in the first period that should be compensated by a tax increase in the second term. Thus, it is not the friction in the financial markets nor the collateral-transfer cost per se that invalidate the RE, but rather how this cost affects the household's indebtedness and budget constraint.

The mechanism surrounding this finding is the credit channel of the fiscal policy. Lenders, under the zero-profit condition, supply credit depending on whether the borrowers will de- fault. Thus, each regime of either default or no-default generates a credit constraint that depends on both the collateral-transfer cost and future value of the collateral. Moreover, the impact of the credit limit on the budget set depends on the relationship between the lump-sum tax and the transfer cost. Whenever the tax cut cannot be entirely offset by the transfer cost, the public debt is not neutral because there is a net tax balance leftover that affects budget set and real allocations in the economy. The RE deviation depends on the relationship between the collateral-transfer cost and the tax-cut enacted by the fiscal policy.

[1] Notice that the classical default definition, which requires that the future value of collateral falls below the face value of the debt, is stronger than ours in the sense that it implies our default condition, but the converse is not true.

Technically, we show that if the collateral-transfer cost is arbitrary, then the public debt is not neutral and the RE fails even under lump-sum taxes. However, if the collateral-transfer cost is an affine function with unit slope in the lump-sum tax, then the neutrality of the public debt and the RE are restored.[2] Nevertheless, this requirement is not robust in the sense that any perturbation in this unique relationship between transfer-cost and tax will imply non-neutrality of the public debt. This result suggests that the RE tends to be more fragile in economies with discretionary transfer costs and complex tax systems, where the public debt might be even more relevant to affect real allocations.[3]

These findings are in line with Abel (1986), Auerbach and Kotlikoff (1987), Feldstein (1988), Evans et al. (2012), Divino and Orrillo (2017), among others, who argue that any deviations from primary assumptions, such as complete markets, rational expectations, and lump-sum taxes, usually invalidate the RE in practice. Likewise, there is a wide literature favoring the RE that is omitted for the sake of shortness. Barro (1976), for instance, is one of the major advocates of the RE by replying to criticisms from Buchanan (1976) and Feldstein (1974). More recently, Barro (2023) claimed that the RE holds even under incomplete markets, provided that the return on equity is greater than the economic growth rate. For comprehensive reviews on this controversial issue, we refer to the surveys by Seater (1985, 1993) and Ricciuti (2003).

The paper is organized as follows. Section 2 presents the GEI economy, represented by the optimization problems faced by borrowers and lenders and the fiscal policy. Section 3 reports and discusses the major findings related to the RE under collateralized debt. Finally, section 4 is dedicated to the concluding remarks.

[2] A similar constraint appears in Panadés (2001), where the RE only holds when the fine charged on the taxpayers is a constant share of the evaded taxes.

[3] We consider only the possibility of private default in the GEI economy. See, for instance, Pouzo and Presno (2022) for a GEI economy where the government can default on its debt. Arellano (2008) addressed the default risk and income fluctuations for emerging economies.

2. The Model

The economy lasts for two periods, with uncertainty only in the second one represented by a finite number of states of nature, $s = 1, 2, \ldots, S$. For the sake of comparison with Hayford (1989), uncertainty is modeled by a finite probability space (Ω, F, P) with $\Omega = \{1, 2\}$ defining the states of nature and $P = (p_1, p_2) >> 0$, with $p_1 + p_2 = 1$, representing the probability distribution.

There are four agents in the economy, represented by identical households, competitive risk-neutral financial intermediaries, government, and an external lender that does not interact with the households. They consume the numeraire good, borrow funds from financial intermediates, who require collateral, and pay taxes to the government. Financial intermediaries operate in a competitive market and lend funds obtained from the external lender. They pay the interest rate i to the external lender and charge the rate r from the households. The former is exogenous while the latter is taken as given by the financial intermediaries. The government borrows from the external lender at the same interest rate, i, as the financial intermediaries and use the money to fund a first-period tax-cut only applied to the households.

2.1. The housing market

Unlike Geanakoplos and Zame (2014), who assume that the collateral transfer is smooth and cost free, we admit that there is a transaction cost in case of default. It is natural to suppose that such a cost is paid in full by the lenders, who execute the collateral to compensate the loss in case of default. However, we will allow for a voluntary agreement between lenders and borrowers to share this collateral-transfer cost.

For the sake of realism, we carry out the analysis for the housing market so that, from now on, the borrower and the lender are considered as mortgagor and mortgagee, respectively. Thus, the mortgagor borrows from the mortgagees to buy a house whose value in the first period, c_o, is assumed to be exogenous. This house is offered as a collateral that will be executed in case of default after the payment of the transfer cost agreed between the involved parts. The future value of the house is assumed to be a random variable $c = (c_1, c_2)$ whose expected value is $E[c] = p_1 c_1 + p_2 c_2$.

The mortgagor is characterized by their utility function $U: R_+^{1+S} \rightarrow R$ assumed to be continuous and strictly increasing, and initial endowment $\omega \in R_+^{S+1}$. A consumption plan $x \in R_+^{1+S}$ is defined by a current consumption x_o and a random variable $x_{-o} = (x_1, x_2)$ representing the next period consumption. Note that x_{-o} is the vector x but dropping the first coordinate.

Our analysis is carried out in loan markets, as in Hayford (1989), and not in asset markets, as in Geanakoplos and Zame (2014). The loan contracts are subject to non-voluntary default, in the sense that the mortgagor will default whenever the net future value of the house (which includes transaction costs) is smaller than the loan claim (interest plus principal). Our debt enforcement mechanism, given by the seizure of collateral, generalizes the relationship between the RE and the compensation scheme, i.e., how large is the transfer cost compared to the tax cut.

2.1.1. The mortgagor problem

The mortgagor's problem is to maximize their utility function[4] $U(x_o + c_o, x_{-o})$ subject to the following budget constraints:

$$x_o + c_o \leq \omega_o + \phi - t_o, \tag{1}$$

$$x_s + z_s \leq \omega_s - t + c_s \tag{2}$$

where $z_s = \min\{c_s - \tau, (1 + r)\phi\}$ and $\tau > 0$ is the cost incurred in the collateral transfer, assumed to be paid by the mortgagee to the mortgagor in the event of default.

Constraints (1) and (2) are the first-and second-period classical budget constraints, respectively. The former states that the mortgagor spends money on consumption and acquisition of house, which are funded by the income after the payment taxes and the mortgage loan. The latter indicates that, in each state of nature, the mortgagor consumes and makes partial or total payments of the loan depending on the comparison between the value of the claim and the value of the house after deducting the transfer cost, τ. All the expenditures are funded by the contingent wealth net of taxes.

[4] In applications, U usually takes the form of an expected utility.

Remark 1. *Although the initial endowments are exogenous, the net income is endogenous due to the possibility of default.*[5] *This is like Panadés (2001), where income is endogenous due to tax evasion, but contrary to Strawczynski (1995), where uncertainty is exogenous.*

We assume a simple relationship between the collateral transfer cost, τ, and the lump-sum tax, *t*, to validate the RE. Then, we investigate how deviations from this naive framework might affect both the neutrality of the public debt and the RE through the credit channel of the fiscal policy. We suppose that τ is a linear affine function of the second-period lump-sum tax, $t > 0$, such that:

$$\tau = f(t) = \alpha t + \beta, \ \forall t, \ \text{with } \alpha \neq 0 \text{ and } \beta \in R_+ \tag{3}$$

The loan ϕ must satisfy the following collateral constraint:

$$0 \leq \phi < c_0 \tag{4}$$

Otherwise, the mortgagor's problem will have no solution, as is well known in the specialized literature. The inequality (4) states that the "haircut" (or the difference between the value of the collateral and the loan) must be strictly positive. This non-arbitrage condition is necessary for the mortgagor's problem to have a solution.6

Remark 2. *The transfer cost τ can be interpreted, for instance, as the monetary value of the "deed in lieu of foreclosure", which formally transfers the ownership of a property from the current owner to the lender in exchange for the mortgage debt.*

2.1.2. The mortgagee problem

The expected profit from lending, $\theta \geq 0$, at the interest rate *r* is given by:

$$\prod(r, \theta) = E[z_s - (1 + i)\theta], \tag{5}$$

where $z_s = \min\{c_s - \tau, (1 + r)\theta\}$, and τ is the collateral-transfer cost defined earlier.

[5] Indeed, the delivery rate z_s, among other things, depends on *r* and ϕ that are endogenous.
[6] This condition is also very common in the housing market, where the loan can fund only a fraction of the house total value.

Since Ω is a finite set, we can assume without loss of generality that $c_1 \leq c_2$. Under this condition and the definition of default, the following might occur:

- Mortgagees suffer a complete default if there is default in state $s = 2$. That is,

$$c_1 - \tau \leq c_2 - \tau < (1 + r)\,\theta \tag{6}$$

- Mortgagees do not suffer any default if there is no default in state $s = 1$. That is,

$$(1 + r)\theta \leq c_1 - \tau \leq c_2 - \tau \tag{7}$$

- Mortgagees suffer a partial default if there is default in at least one state of the nature. That is,

$$c_1 - \tau < (1 + r)\,\theta \leq c_2 - \tau \tag{8}$$

2.2. *Fiscal policy*

The government enacts a tax cut in the first period and a tax increase in the second one to balance the public budget. That is to say,

$$dt_o = -d;\; dt = (1 + i)\,d = -(1 + i)dt_O \tag{9}$$

where $d > 0$ is the debt issued to fund the tax-cut and $(1 + i)$ is the gross interest rate paid by the government to the lender.

2.3. *Equilibrium*

The economy is said to be in equilibrium if all agents' choices are optimal and the loan market is cleared, $\theta = \phi$. By optimal choices we mean that the mortgagor maximizes their utility function subject to budget and borrowing constraints, the mortgagee maximizes profits, and the government balances its budget according to the fiscal policy given by (9). The rationality behind this fiscal policy to balance the government's budget is explained in the Appendix.

3. The Ricardian Equivalence

This section reports and discusses the major results on the RE under collateral constraints, whose proofs are referred to the Appendix. As a preliminary finding, Lemma 1 deals with the supply of credit in the collateralized economy.

Lemma 1. *Assume that $c_1 < c_2$. Then, under the zero-profit condition for mortgage lenders, the loan schedule θ satisfies the following:*

$$1 + r = \begin{cases} 1 + i & if\ \ 0 < \theta \leq \theta_1 \\ \dfrac{1+i}{p_2} - \dfrac{p_1}{p_2}\dfrac{c_1 - \tau}{\theta} & if\ \theta_1 < \theta \leq \theta_2 \end{cases}$$

where $\theta_1 = (c_1 - \tau)/1 + i, \ \ \theta_2 = (E[c] - \tau)/1 + i.$

Proof. See the Appendix.

Figure 1 illustrates the shape of the credit supply derived according to Lemma 1

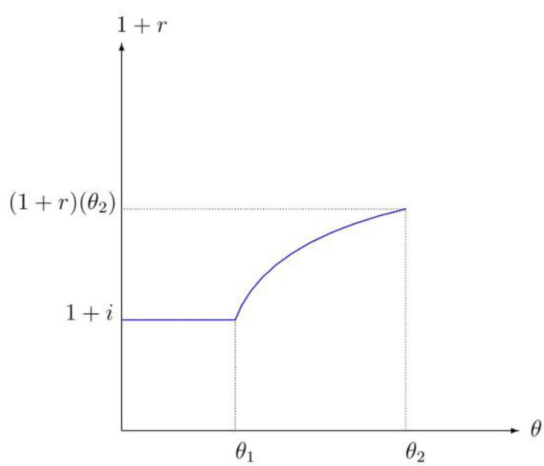

Figure 1 - Loan schedule according to specifications and boundaries reported in Lemma 1.

Remark 3. *It is worth stress that the credit limits θ_1 and θ_2 of Lemma 1 depend on the collateral-transfer cost τ, which in turn is an affine function of the lump-sum tax, t, according to equation* (3). *This interdependence allows the households to neutralize the tax-cut given by equation* (9) *under a specific parametrization. It represents the credit channel of the fiscal policy.*

Remark 4. *The proof of Lemma 1, shown in the Appendix, encompasses the cases of full-, partial- and no-default by the borrowers in the economy.*

The next Lemma formalizes the relationship between the credit supply and the tax cut (or tax increase).

Lemma 2. *Let θ_1 and θ_2 be given as in Lemma 1 and τ as in equation* (3). *If $c_o > \frac{E[c]-\tau}{1+i}$ and the government enacts a fiscal policy represented by equations* (9) *and* (6), *then:*

$$d\theta_j - dt_o = (\alpha - 1)dt_o = (1-\alpha)d\,t/(1+i)\,, j = 1,2 \tag{10}$$

Proof. The proof is straightforward.

The Lemma 2 relies on the credit channel of the fiscal policy. More precisely, it states how the households' indebtedness could neutralize a tax-cut fiscal policy. If the variation rate of the collateral transfer cost with respect to the lump-sum tax is different from one ($\alpha \neq 1$), then the tax cut will affect the households' budget constraint and so the real allocations in the economy. Otherwise, if $\alpha = 1$, there will be no effect. In this environment, we can derive the following result for the RE.

Proposition 1. *Under the assumptions of Lemma (2), the following statements are true:*

The RE does hold if $\alpha = 1$ *in equation* (3).

The RE does not hold if $\alpha \neq 1$ *in equation* (3).

Proof. See the Appendix.

The condition $\alpha = 1$ is not robust because any small-enough perturbation in the parameter α will make the economy lies in item 2 of Proposition 1, which is robust. Thus, only under the very specific condition of $\alpha = 1$ the public debt is neutral and the RE holds in this economy with default and collateral constraints. This result makes the occurrence of the RE somewhat unlikely in practice, even under remarkably favorable technical conditions provided by such a naive specification. Upon default, only the net value of the collateral being transferred to creditors gives rise to equilibrium limits that depend on taxes, since by assumption the collateral transfer cost depends on taxes as well.

A novelty here is the transmission of the fiscal policy through the credit channel. If the collateral-transfer cost is higher than the tax cut, there will be a reduction in consumption in case of default because the collateral seizure would result in a positive net tax payment. Otherwise, if the transfer cost is smaller than the tax cut, there will be an increase in consumption because the event of default would be followed by a negative net tax payment. In both cases, real allocations would be affected by the occurrence of default in the collateralized economy. Only in the special case where the tax cut exactly offsets the collateral transfer cost upon default there will be no impact of the fiscal policy on the real allocations and the RE holds.

Conceptually, the contribution of Proposition 1 is fundamentally different from the one that would have emerged in the case of liquidity constrained households because the credit channel of the fiscal policy is the only one at work due to the presence of transaction costs in the collateralized economy. If not fully compensated, this cost imposes a constraint to the households that resembles a liquidity constraint and ultimately invalidates the RE.

4. Concluding remarks

This paper investigated conditions for the Ricardian Equivalence (RE) to hold in a GEI economy with default and collateral constraints. The analysis focused on the housing market, where mortgagors and mortgagees might voluntarily enter in an agreement of good faith to defray the collateral--transfer cost in the event of default. The government charges lumpsum taxes from the mortgagors and enacts a first-period tax cut that is compensated by future tax increases. Only under a very specific case where the tax cut exactly offsets the collateral-transfer cost there is evidence of neutrality of the public debt and the RE holds in the economy. The debt enforcement mechanism illustrates how the RE depends on the compensation scheme, i.e., how large is the transfer cost compared to the tax cut.

In this environment, key arguments for the RE to hold or to fail rest on the following facts. Firstly, if there is default in only one state of nature, the inverse supply of credit is strictly increasing in a non-degenerated interval. Secondly, under a zero-profit condition for the financial intermediaries, the endpoints of the previous interval (the credit supply) are equal to the present value of the collateral minus the collateral-transfer cost. Thirdly, in equilibrium, the supply of credit determines the mortgagor's budget set. Finally, since the collateral is exogenous and the budget constraint is binding,[7] the mortgagor will be able to neutralize the public debt depending solely on the relationship between the collateral-transfer cost and the second-period lump-sum tax, as reported in Lemma 2 and Proposition 1.

This interdependence among credit limit, transaction cost and lump-sum tax fully characterize the credit channel of the fiscal policy. For the RE to hold in this economy, the collateral-transfer cost must be a linear affine function with unit slope in the second-period lump sum tax. Any deviation from this primary condition will make the public debt non- neutral and the RE to fail. As this condition is not robust, even under a naive framework, one might expect the fiscal policy to affect real allocations through the credit channel as the general case, especially in economies with complex tax systems and collateral transaction costs.

[7] Due to the strict monotonicity of the preferences.

Appendix

Proof of Lemma 1

Proof. Using (6), (7) and (8), the mortgagee's expected profit is explicitly defined as:

$$\Pi(\theta, r) = \begin{cases} (1+r)\theta - (1+i)\theta & if\ (1+r)\theta \le z1 \\ p_2(1+r)\theta + p_1(c_1 - \tau) - (1+i)\theta & if\ z_1 < (1+r)\theta \le z_2 \\ E[z-(1+i)\theta] & if\ z_2 < (1+r)\theta \end{cases}$$

In the first case, the borrower never defaults, since $c_1 \le c_2$. In the intermediate case, the borrower only defaults in one state, namely in $s = 1$, where the delivery is $z_1 = c_1 - \tau$. In this case, we have that $c_1 < c_2$. In the last case, the borrower always defaults, since $c_2 \ge c_1$.

If $\theta > 0$, the zero expected profit condition implies that $1 + r = i + 1$, and the maximum θ denoted by θ_1 satisfies $(1 + r)\ \theta_1 = z_1$.Thus, $\theta_1 = (c_1 - \tau)/1 + i$. For the intermediate case, it follows from the zero expected profit condition that $1 + r = \frac{1+i}{p_2} - \frac{p_1}{p_2}\frac{c_1-\tau}{\theta}$ for every $\theta > \theta_1$, where the maximum θ denoted by θ_2 satisfies $E[z] = (E[c] - \tau) = (1 + i)\ \theta_2$ once $z_2 = (1 + r)\theta$. For the last case, lenders will not supply any credit because they know that borrowers will always default. Thus, Lemma 1 follows.

Proof of item 1 of Proposition 1

Proof. Define:

$$\underline{x}_o = w_o + \theta_1 - t_o - c_o \ \ and\ \overline{x}_o = w_o + \theta_2 - t_o - c_o,$$

where $\theta_1 = \frac{c_1-\tau}{1+i}$ e $\theta_2 = \frac{E[c]-\tau}{1+i}$

For $\theta \in\ ]0, \theta_1]$, one has that $x_o = w_o + \theta - t_o - c_o \le w_o + \theta_j - t_o - c_o$, with $j = 1, 2$. Therefore, $x_o \in\ ]0, \underline{x}_o]$. Now, for $\theta \in\ ]\theta_1, \theta_2]$, one has that $x_o \in]\ \underline{x}_o, \overline{x}_o\ ]$. So, $(0, \underline{x}_o] \cup (\underline{x}_o, \overline{x}_o]$ provides a feasible set for the first-period consumption, x_o.

From Lemma 2, clearly we have that both $d\underline{x}_o$ and $d\overline{x}_o$ are equal to $(\alpha-1)dt_o$ or $(1-\alpha)\frac{dt}{1+i}$. In either case, if $\alpha \neq 1$, we have that both $d\underline{x}_o$ and $d\overline{x}_o$ are different from zero. This is sufficient to show that the RE does not hold.

Proof of item 2 of Proposition 1

Proof. From item 1 of Proposition 1, it follows that if $\alpha = 1$, then both $d\underline{x}_o$ and $d\overline{x}_o$ are equal to zero. Next, we analyze the second-period consumption for each state $s \in \Omega$. By using budget constraint (2), one has that:

$$x_s = \omega_s - t - z_s + c_s$$

We consider two cases for $\tau = t + \beta$, with $\tau - t = \beta \geq 0$

Case 1: $c_1 - \tau \geq (1 + r)\phi$. Here, there is no default because we are assuming $c_1 < c_2$.

Since $c_o > \frac{E[c]-\tau}{1+i} > \frac{c_1-\tau}{1+i}$ the maximum borrowing is $\phi = \theta_1$ if there is no default. Thus,by using the fact that $1 + r = 1 + i$, one has that:

$x_1 = \omega_1 - t - (1+r)\ \theta_1 + c_1 = \omega_1 + \beta$, once that $\theta_1 = \frac{c_1-\tau}{1+i}$.

Therefore, $dx_1 = 0$.

For $s = 2$, it follows that the maximum borrowing is $\phi = \theta_2 = \frac{E[c]-\tau}{1+i}$. However, in this case, we also have that $1 + r = 1 + i$. Hence:

$x_2 = \omega_2 - t - (1 + r)\theta_2 + c_2 = \omega_2 - E[c] + c_2 + \beta > 0$,because $c_1 < E[c] < c_2$.

Then, $dx_2 = 0$ because collateral and probabilities are exogenous.

Case 2: $c_1 - \tau < (1 + r)\phi \leq c_2 - \tau$ and there is default in state 1. Thus:

$$x_1 = \omega_1 - t - (c_1 - \tau) + c_1 = \omega_1 + \beta \text{ which implies that } dx_1 = 0$$

For state 2, where there is no default, we have that:

$$x_2 = \omega_2 - t - (1 + r)\phi + c_2.$$

From Lemma 1, it follows that:

$$x_2 = \omega_2 - t - \left(\frac{1+i}{p_2} - \frac{p_1}{p_2}\frac{c_1 - \tau}{\theta}\right)\theta + c_2, with\ \theta \in]\theta_1, \theta_2]$$

In this case, the maximum borrowing that the mortgagor can get is $\phi = \theta_2 = \frac{E[c]-\tau}{1+i} < c_o$.

Substituting θ_2 in the previous equation and using the fact that $1 - p_1 = p_2$, we obtain:

$x_2 = \omega_2 + \beta$, which implies that $dx_2 = 0$.

This completes the proof of Proposition 1.

Fiscal policy rationality

The fiscal policy defined by (9) was formulated in terms of a tax-cut in the first period and a tax increase in the second one. To address its rationality, suppose that the government is characterized by its spending $G = (g_0, g\mathbf{1}) \in R^{1+S}_{++}$ with $g\mathbf{1}$ being a vector of R^S_+ whose coordinates are all equal to $g > 0$.

Assume that the government's budget was balanced before enacting the tax cut. That is, $G_0 = g_0 = t_0$ and $G_s = g = t$. The tax cut, however, yields a deficit, such that $g_0 > \tilde{t}_0$. Therefore, to balance the budget set, the government must fund the deficit $\tilde{t}_0 - g_0 < 0$ by issuing bonds. Thus, $\tilde{t}_0 - g_0 + d = 0$. Given that $g_0 = t_0$, then one has that

$$\tilde{d}t_0 + d = 0 \qquad (11)$$

once $dt_0 = \tilde{t}_0 - t_0$.

Because of the first-period tax increase, the government makes a surplus $\tilde{t} > g$, which will pay for the interest rate on the debt d. Thus, one has that $\tilde{t} - t = (1+r)d$. But $g = t$, then implies that

$$dt = (1 + r)d \tag{12}$$

once $dt = \tilde{t} - t$.

For the Ricardian Equivalence, what matters are (11) and (12). These facts were summarized by equation (9) in the main text.

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