Artigos

Divisia monetary aggregates for Brazil*

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ABSTRACT

Herein we review the methodology for constructing monetary aggregates using Divisia weights. Divisia monetary aggregates (M2, M3, and M4) were then calculated with both monthly and quarterly data for the period Jan. 1980-Aug. 1998 and were found to have a better general performance than the corresponding traditional monetary aggregates constructed by the simple-sum of assets. This was so according to the following criteria: a) a better goodness-of–fit coefficient in a regression between the inflation rate and changes in the monetary aggregate; b) a relationship between these two variables which was in most cases closer to proportionality; c) more stable monetary multipliers; and, finally, d) a higher linear correlation coefficient between income velocity and the interest rate.

Key words: divisia aggregates, monetary aggregates, monetary services, monetary stock, money and prices, monetary multipliers.

RESUMO

Neste estudo apresentamos inicialmente um breve resumo da metodologia de cálculo dos agregados monetários com o índice de Divisia.Os agregados monetários (M2, M3 e M4) de Divisia foram então calculados para o período de janeiro de 1980 a agosto de 1998, com dados mensais e trimestrais. Os resultados, em geral, apresentaram-se melhores do que aqueles dos correspondentes agregados monetários tradicionais obtidos da simples soma de ativos, a saber: a) melhor precisão no ajustamento da regressão entre a taxa de inflação e a taxa de variação do agregado monetário; b) relação entre estas duas variáveis que foi, na maioria dos casos, mais próxima da proporcionalidade; c) multiplicadores monetários mais estáveis; e, finalmente, d) maior coeficiente de correlação linear entre a velocidade-renda da moeda e a taxa de juros.

Palavras-chave: agregados de divisia, agregados monetários, serviços monetários, estoque monetário, moeda e preços, multiplicador monetário.

JEL classification: E49, E52, E59.

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1 Introduction

Relatively more attention has been given in the literature to the question of which monetary aggregate (i.e., M1, M2, or any other broader monetary aggregate) could better measure liquidity than to the subject of how properly one should add the assets of a given monetary aggregate. In fact, for the latter the technically unjustifiable simple-sum of assets has been the commonly adopted method. Objections to such a procedure have, however, been raised by some academic quarters for some time, and the proper way to add assets with distinct liquidity has already been known for over twenty years now. It is thus surprising, if not unfortunate, that practitioners around the world, in particular the Central Banks, have largely ignored such research efforts.[1]

For just a brief historic look at the subject of aggregation, Fisher (1922) was probably among the first to call attention to the shortcomings of the simple-sum procedure. More specifically, he noted that in aggregation theory a quantity index should measure the income effect of a relative price change, but not its pure substitution effect (the effect that maintains an individual along the same indifference curve). In fact, the substitution effect must be internalized by the index. An economic aggregate which is a simple-sum of its components can not separate these two effects, unless such components are perfect substitutes.[2]

As the various assets of a given monetary aggregate are, in general, far from being perfect substitutes, the simple-sum procedure is particularly questionable in monetary aggregation. This fact has motivated the proposal of a few alternative schemes for weighting financial assets.[3] Barnett (1990) has argued though that only one among these has a sound microeconomics basis, namely, the Divisia aggregation (Divisia, 1925), which weights assets according to their respective opportunity cost (defined below).[4]

The calculation of Divisia monetary aggregates for Brazil is the main goal of this study. The question of how properly to aggregate financial assets is discussed in the next section where

1 According to Amderson, Jones and Nesmith (1997a) Central bank studies which use Divisia aggregates include countries such as Germany, Japan, Canada and the United Kingdom, but Divisia monetary indexes are only calculated by the Bank of England and the Federal Reserve Bank of St. Louis. In Brazil, the Divisia index is regularly calculated by IPEA (see Boletim de Conjuntura, IPEA, Rio).

2 On these points see Barnett, Fisher and Serletis (1992).

3 Friedman and Shwartz (1970) were among the first to suggest that assets should be weighted according to their moneyness.

4 To mention just one of such alternative weighting schemes, Spindt (1985) suggests weighting the financial assets according to their respective turnover rates. According to Barnett (1990) the use of the turnover rates is, however, totally arbitrary. This is not so with the use of the opportunity cost as weights, for it is obtained after a consumer's optimization process.

we briefly deal with: i) the integration between the economic theory of aggregation and the statistical theory of index numbers; ii) the specific problem of monetary aggregation; and iii) the concept of an economic money stock in order to show how much impropriety there is in the use of the traditional aggregates as a measure of money stock. In Section 3 we apply the Divisia monetary aggregation methodology to Brazilian data. Final remarks are in Section 4.

2 The divisia monetary aggregation: a brief summary

The Divisia index used as a quantity aggregator can be derived from microeconomic principles with practically no restrictions imposed on the individual's utility function, as only the assumption of a linearly homogeneous utility function is required. Suppose we wish to aggregate the quantities of a set with n goods, whose price and quantity vectors are, respectively, given by $p = (p_1, p_2,\dots,p_n)$ and $q = (q_1, q_2,\dots,q_n)$. By maximizing the utility function g(q) subject to the budget restriction

$$\sum_{i=1}^{n} p_i q_i = g(q).f(p) = E, \qquad (1)$$

where f(p) is a price aggregator function and E is total expenditure one obtains from the first order maximization conditions:[5]

$$g(t) = \exp(\int \sum s_i d \ln q_i + c) \qquad (2)$$

where $s_i = (p_i q_i / E)$ is the expenditure share of item i. This is the continuous-time version of the Divisia index which (as it is first-degree homogeneous) is thus an exact aggregator for any utility function. A discrete approximation to this index, also known as the Tornqvist-Theil formula, is given by:[6]

$$\ln g(t) - \ln g(t-1) = \sum \bar{s}_{it} \left(\ln q_i(t) - \ln q_i(t-1)\right), \qquad (3)$$

5 As by assumption the aggregator function is linearly homogeneous, it then satisfies Euler's condition which makes the proof of this result simple. For details see Yue and Fluri (1991).

6 In fact, as noted by Barnett (1990), this is mathematically the Simpson's rule approximation to the continuous-time Divisia index.

where $s^{\bullet}_{it} = \left(\frac{s_{it} + s_{i,t-1}}{2}\right)$ is the average of the expenditure shares of two periods.

Although this discrete-time formula is not exact for any arbitrary aggregator function, Diewert (1976) has shown it is though exact for a linearly homogeneous translog specification, which is itself a second-order approximation to any arbitrary utility function, g(q). Thus, it belongs to what he calls a class of "superlative" index numbers.[7]

By let m_{it} standing for the value of asset i belonging to monetary aggregate M_t in period t, and p_{it} the price of such an asset, the Divisia index in equation (3) would be

$$\ln M_t - \ln M_{t-1} = \sum_{i=1}^{n} s^*_{it} (\ln m_{it} - \ln m_{i,t-1}) \tag{4}$$

where $s^*_{it} = (s_{it} + s_{i,t-1})/2$ and $s_{it} = p_{it} m_{it} / \sum_{k=1}^{n} p_{kt} m_{kt}$.[8]

Thus the formula gives only the monetary aggregate rate of change, which is a weighted average (the weights are the expenditure shares) of the change rates of the assets belonging to the monetary aggregate.[9]

With Diewert's (1976) contribution it became possible to combine the economic aggregation theory with the statistical theory of index numbers. That said, as any index-number formula would require the use of both prices and quantities, a remaining question was then how to attribute a price to an asset. A solution to this problem was independently proposed by Barnett (1978) and Donovan (1978), namely that the price of an asset ought to be its opportunity cost. As shown by Donovan (1978), one should proceed in this area similarly to the case of a dura-

7 Fisher's Ideal index also belongs to the class of superlative indexes, since it is exactly correct for a utility function g(q), which is the square root of a quadratic specification. The popular Paasche and Laspeyres indexes, by providing only first-order approximations to an unknown aggregator function would not qualify as a superlative index number, however.

8 As discussed below in the text, the value of a monetary aggregate (and its assets) can be used here either in nominal or real terms.

9 The Divisia index can obviously also be written as $M_t = M_{t-1} \prod_i (m_{it} / m_{i,t-1})^{(s_{it} + s_{i,t-1})/2}$ This was the formula actually used in this study.

ble good. That is, a price would have to be imputed to the service flow produced by the use of that good (or asset). He has shown such an opportunity cost (rental price or user cost) to be:[10]

$$p_{it} = p_t^*(R_t - r_{it})/(1+R_t), \tag{5}$$

where r_{it} is the nominal interest rate for asset i (which is zero in the case of currency), p_t^* is a general consumer price index, and R_t is the maximum interest rate in the economy (its only function is the intertemporal transfer of wealth with no concern regarding the liquidity of the asset). This is the nominal opportunity cost which, when divided by the general consumer price index, gives the real opportunity cost. In fact, when calculating the expenditure share for the Divisia monetary aggregate one can use either the asset's real opportunity cost times its nominal value or the asset's nominal opportunity cost times its real value, as they will both give the same result.[11] Furthermore, in the actual implementation of the Divisia index formula one can simply multiply the asset's nominal value by $p_{it} = R_t - r_{it}$, for when calculating the expenditure shares the denominator of equation (5) cancels out.

Although the Divisia index discussed above gives only a flow of monetary services, Barnett (1978) has also shown how to obtain a proper measure of an economic stock of money. We present this point here particularly to show how much impropriety there is in the use of the traditional simple-sum aggregate as a measure of a stock of money. More precisely, by combining the consumer's multi-period budget constraint into a single budget constraint the term which represents the economic stock of money (or stock of monetary wealth) is

$$V_t = \sum_{s=t}^{\infty} \sum_{i=1}^{n} \left[\frac{R_s - r_{is}}{\prod_{u=t}^{s}(1+R_u)} \right] m_{is} \tag{6}$$

As this gives the discounted present value of all current and future expenditure on monetary services it can properly be interpreted as a stock of monetary wealth. Its measurement is diffi-

10 In Barnett (1978 and 1980), one finds a formal presentation of this result. Essentially, the rental price is derived from microeconomics principles where a representative consumer makes an optimal intertemporal choice between his consumption pattern and his financial asset portfolio.

11 On these points see Anderson, Jones and Nesmith (1997b and 1997c).

cult, however. But if one is willing to assume static expectations for the representative consumer that equation can be reduced to (Barnett and Thou, 1994):[12]

$$V_t = \sum_{i=1}^{n} \frac{R_t - r_{it}}{R_t} m_{it}, \tag{7}$$

which can now be easily implemented.

By noting that an asset which pays interest rate might at the same time provide some monetary services, Barnett and Zhou (1994) then proposed separating the part of the asset which represents the investment return from the one representing the asset's monetary service. Specifically, they first defined the present and future interest received on monetary assets, discounted to time t, as

$$V_t^* = \sum_{s=t}^{\infty} \sum_{i=1}^{n} \left[\frac{r_{st} m_{st}}{\prod_{u=t}^{s} (1+R_u)} \right] \tag{8}$$

which by assuming, once again, static expectations for the representative consumer can be reduced to

$$V_t^* = \sum_{i=1}^{n} r_{it} m_{it} / R_t \tag{9}$$

It turns out that by adding equations (7) and (9) one gets $V_t + V_t^* = \sum_{i=1}^{n} m_{it}$ which is the traditional monetary aggregate obtained by the simple-sum of assets. Thus, such a monetary aggregate would only qualify as a proper measure of a stock of monetary wealth when the discounted present value of its interest income is null. The latter term can then be interpreted as an error-in-variable component of the traditional simple-sum aggregate in case it is used as a measure of monetary wealth. It follows that when a monetary aggregate is defined to include

12 This equation was first shown by Barnett (1991) to be the same as the currency-equivalent (CE) index proposed by Rotemberg (1991), which is also discussed in Rotemberg, Driscoll and Poterba (1995).

only currency and demand deposits, both of which do not pay any interest rate (at least as used here), it then qualifies as a proper stock of monetary wealth; notice that in such a case we actually have $V_t = \sum_{i=1}^{n} m_{it}$, as $V_t^* = 0$.[13] The error-in-variable components for aggregates M2, M3 and M4, as calculated for Brazil, are discussed below.

3 Data and results

We use here Brazilian data to compare the performance of the simple-sum and the Divisia weights of assets in a number of situations. The data cover the period 1980/1998, thus including a variety of plans of economic stabilization, such as Cruzado Plan (March 1986), Collor Plan (February 1990) and Real Plan (June 1994). The monetary aggregates considered where those published by the Central Bank and are defined as following: M1 = currency plus demand deposits; M2 = M1 plus government bonds held by the public; M3 = M2 plus savings deposits; and M4 = M3 plus time deposits. As for the interest rates: i) a zero rate was assumed for the assets part of M1; ii) the overnight rate was used for government bonds; iii) the interest rates for savings were those uniformly paid by the private commercial banks, but which are set by the government; and, finally, iv) the interest rate for the Certificate of Banking Deposits (CDB) was used for the time deposits.[14]

As discussed in connection with equation (4), a major difficulty in calculating an asset rental price is the choice of the maximum interest rate in the economy, R. In this study, which uses both monthly and quarterly data, two alternative criteria were used in the choice of R. By the first of these, a maximum value was chosen among the interest rates of the various assets of a given monetary aggregate, and then it was multiplied by 1.05 so that the rental price share of each asset would be positive.[15] By the second criterion, the largest value among the following

13 This discussion suggests that an appropriate aggregate should probably include many more assets than the ones presently being considered officially as part of a monetary aggregate. Assets such as those represented by some investment funds, which can easily be converted to currency (or demand deposits), certainly contribute to the liquidity of the economy. In this connection, Barnett and Zhou (1994) showed that by adding bond and stock mutual funds to USA aggregate M2 one can actually reduce the variability of the income investment (non-moneyness) of such an aggregate.

14 It is sometimes suggested that as the demand deposits, distinctly from currency, do receive a sort of an implicit interest rate (perhaps in the form of special loans to the depositors), then a distinction should be made between these two assets. Due to the difficulties in the actual estimation of such an implicit rate in the case of Brazil, however, we treated the two assets as identical here.

15 It should be mentioned here that in Rossi and Silva (1991a), which covers a shorter period than the one of this study, only minor changes in results were found when the multiplying factor was instead 1.10.

items was chosen: i) the interest rates of the various assets part of a given monetary aggregate, ii) the rate of change of the stock market index of both Rio de Janeiro (IBVR) and São Paulo (IBOVESPA), and iii) the rate of change of the exchange rate of the parallel (black) market (R$/US$).[16] The results by the two criteria differ to some extent, with the second criterion given slightly better results, as we shall see.[17]

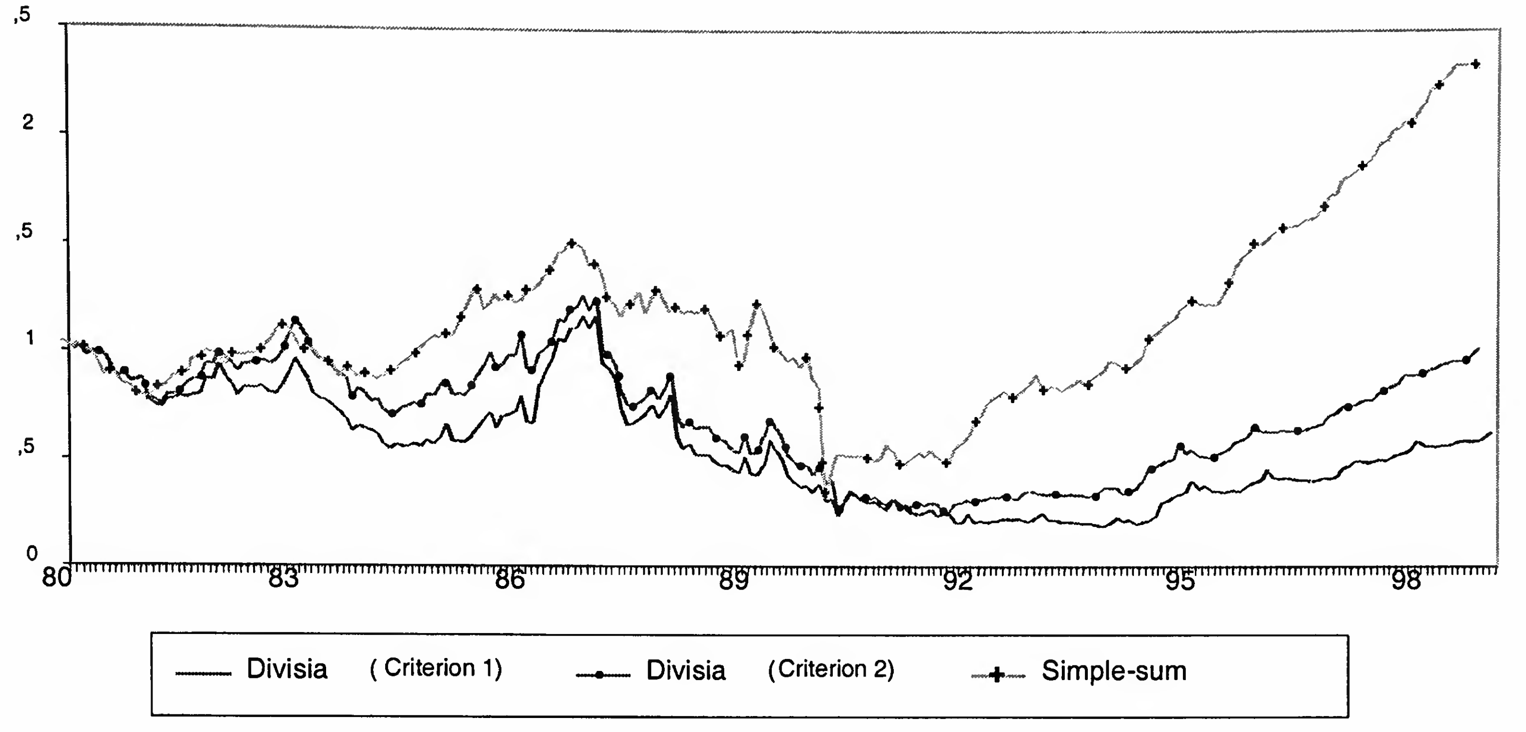
Since among all the monetary aggregates M4 is the one with the largest number of assets, including some with not much liquidity, then a comparison between the Divisia and the simple-sum procedures would, in this case, be all the more relevant as this will exaggerate liquidity as defined by the latter. Thus, in the following analysis relatively more attention will be paid to the comparisons involving such a monetary aggregate. With this in mind, we plot in Figure 1 monetary aggregate M4, calculated with monthly data, for both Divisia weights (using the two criteria in the choice of R) and simple-sum of assets. As it can be seen, liquidity increased much more with the simple-sum than with the Divisia procedure. Notice also that the curve for the Divisia aggregate obtained by criterion 2 is always above the one obtained by criterion 1. The difference between these two measures diminishes, however, near to the Collor Plan (March 1990) when their values were practically identical. Afterwards, the two results start diverging once again.[18]

16 Of course, one can think of alternative criteria on this matter. For instance, one of the referees suggested that in the case of the stock market index a risk premiun rate should be added.

17 It is interesting to compare these results with those obtained by a few other studies. For instance, Barnett and Spindt (1982) concluded, after a series of experiments, that the Divisia index was robust to changes in R, which were within plausible values. Chou (1991) in an application to USA data concluded, however, that the Divisia index was sensitive to the criterion used in choosing R. This was also the case in Rossi (1993) which, by using a shorter period than the one of the present study also concluded that the results changed somewhat with alternatives use of R. Notice that these conclusions are not in conflict with the previous note which deals, in fact, with just one criterion in the choice of R, as in that case R is merely multiplied by two distinct constants.

18 As similar considerations apply to the aggregates M2 and M3 we omit here the analysis on this point concerning such aggregates.

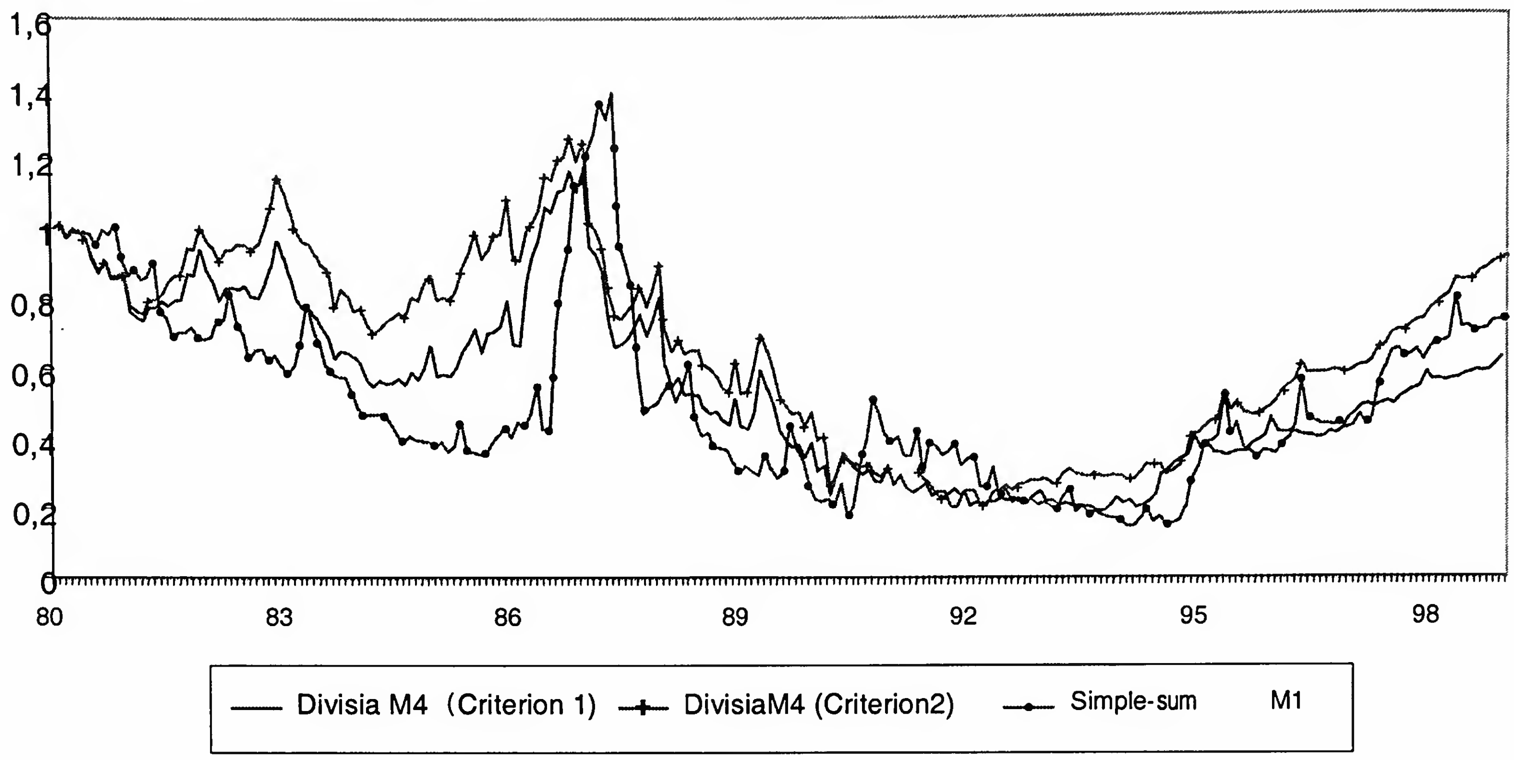
Figure 1
Real Monetary Aggregate M4



In Figure 2 we also compare the results of the Divisia monetary aggregate M4 (calculated by criteria 1 and 2) with those of the simple-sum monetary aggregate M1. It is interesting to note that the results for the Divisia M4 are much closer to the simple-sum M1 than to the simple-sum M4. This should be no surprise. Recall that M1 contains basically the most liquid assets, while M4, besides these same assets, also includes some assets much less liquid. Thus, when the latter are added by the simple-sum to the former the result will certainly be larger than that obtained by Divisia weights, which takes into account only the asset's liquidity. In fact, when calculating the Divisia M4 the expenditure share (not shown here) for the assets represented by currency and demand deposits is by far the largest. As a consequence, the Divisia M4 will behave much like the simple-sum M1.[19]

19 For a more detailed analysis of these expenditure shares, see Rossi (1993).

Figure 2
Real Monetary Aggregates (Divisia M4 *versus* Simple-sum M1)



In spite of a general similarity between the Divisia M4 and simple-sum M1, it is interesting to note that during the stabilization plans (Cruzado, Collor and Real, respectively, 1986, 1990 and after 1994) the latter changed much more than the former (see Figure 2). This is not difficult to explain though. Recall that the expenditure shares used by the Divisia weights are, in fact, averages of the expenditure shares of two neighboring periods. That being the case, any sudden change in the demand for a given asset would have its effects diluted into two periods. Note that here, unlike the previous figure, the simple-sum aggregate M1 can sometimes be above the Divisia aggregate M4. There is nothing unusual about such an outcome, for it depends upon the behavior of the expenditure shares which, in the case of the Divisia M4, can change much over time, as shown in Rossi (1993).

The confiscation of assets during the Collor Plan (February 1990) is a good study case for comparing the performance of the two alternative aggregating procedures. First, notice, by returning once again to Figure 1, the simple-sum M4 fell much more than the Divisia M4.[20] This is basically because of the following: between February and March of 1990 the stock of government bonds fell by about 70%.[21] While this would have a major impact on the simple-sum

20 The analysis in this paragraph is similar to the one presented in Rossi (1993).

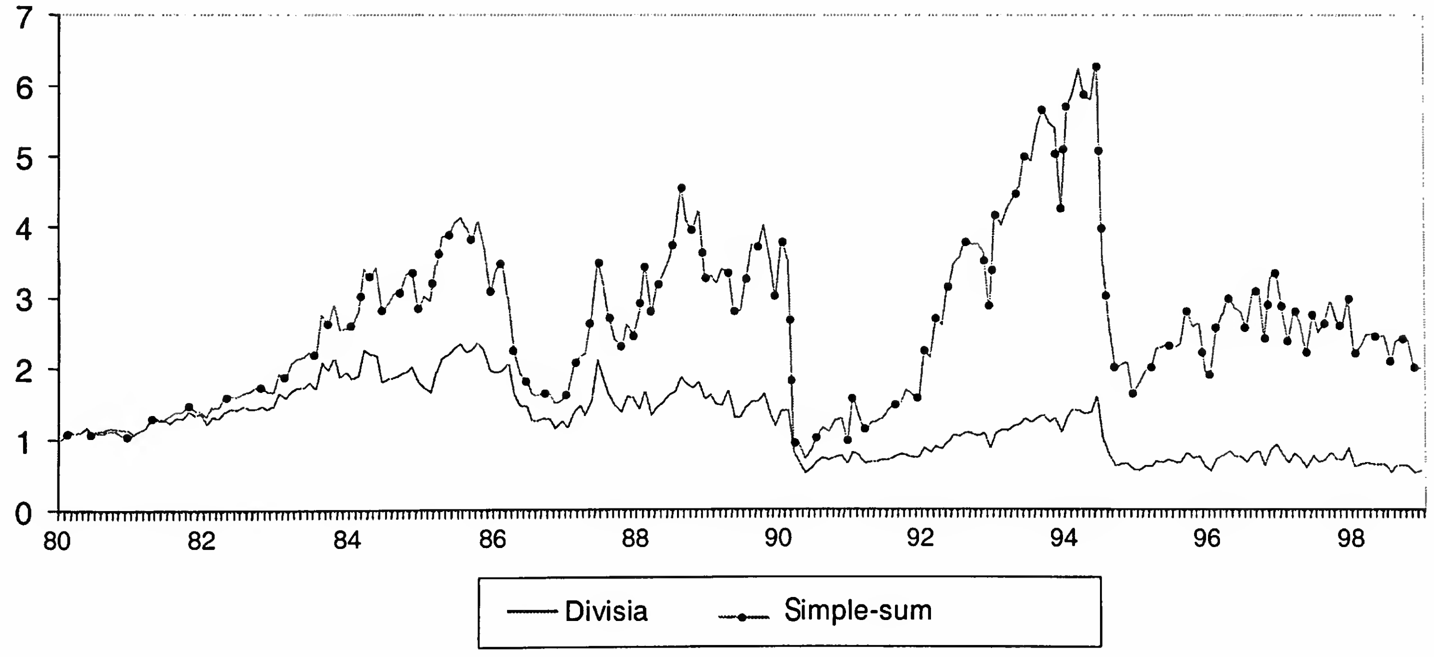
21 For this number see Rossi and Silva (1991b).

M4, its effect would be much smaller in the case of Divisia aggregates M2, M3 and M4. This is so for at least one reason: in view of its high interest rates government bonds would, in such a case, contribute less to liquidity having then little effect on the Divisia results.[22]

We analyze now the behavior of the monetary multipliers. Figure 3A shows the results of the simple-sum and Divisia weights for aggregate M4. As the latter has a smaller variance one might be tempted to conclude that liquidity measured by Divisia weights could be more precisely controlled by the Monetary Authority. Yet the fact is the Divisia index merely indicates the result of a weighting procedure of financial assets which happen to be part of a given monetary aggregate. As such, its controllability depends upon how controllable those assets are. And, as is well known, the Monetary Authority can reasonably control only the monetary base. After all, the broader monetary aggregates would depend upon the behavior of both the individuals and the banking sector concerning their choices of assets, which is not easily predictable.

Figure 3B shows similar multiplier comparisons but between Divisia M4 and simple-sum M1. Now the patterns of the two multipliers are for most part similar. In fact, in view of Figure 2, this result was to be expected, for as that figure shows, there is a closer relationship between Divisia M4 and simple-sum M1 than the one between the Divisia M4 and simple-sum M4 (see Figures 1 and 2).[23]

Figure 3A
Monetary Multipliers for Aggregate M4 (monthly data)

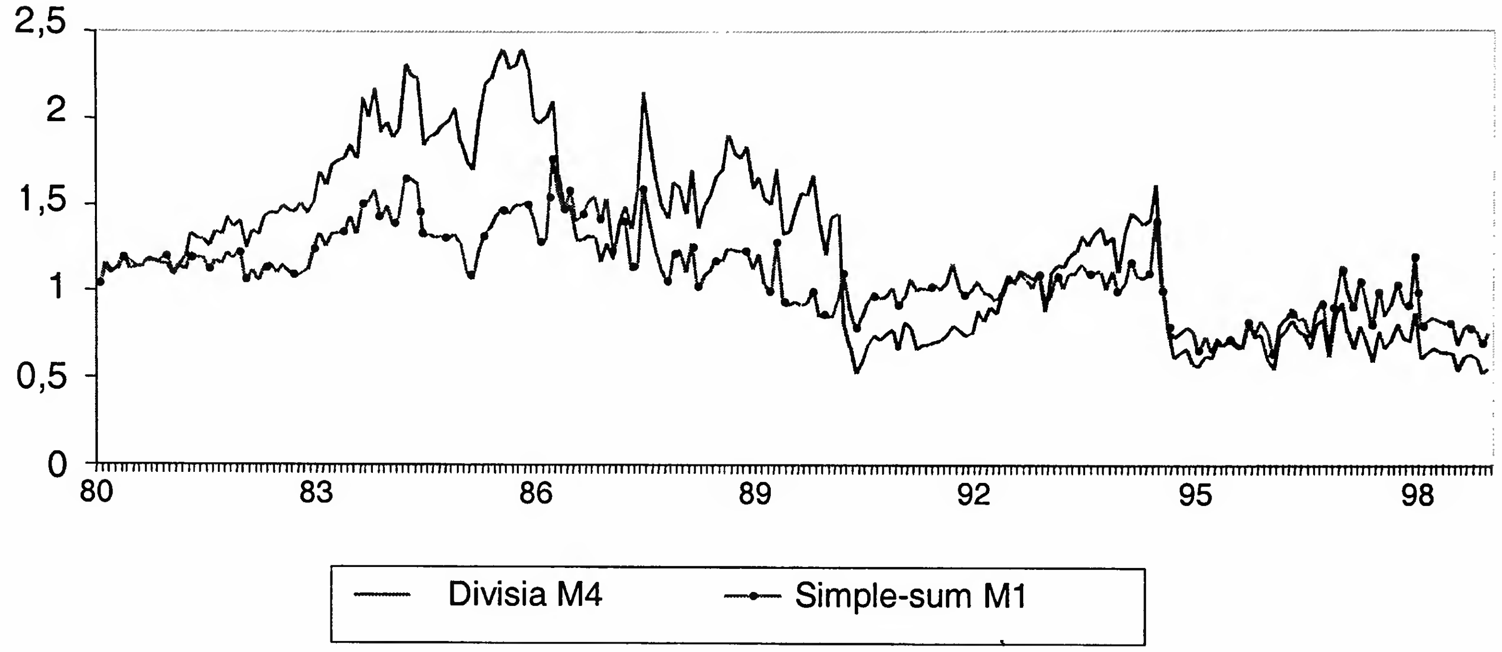


22 Figures 1 and 2 also show a pronounced increase in liquidity following the Cruzado Plan in February 1986, and this was so with either the Divisia M4 or the simple-sum aggregates M1 and M4.

23 It was also found that for aggregate M4 velocity is more stable using the Divisia than the simple-sum procedure, while the velocities for Divisia M4 and simple-sum M1 are very close to each other, which in view of Figure 2 was actually to be expected.

Another point of interest is a comparison of the relationship between the income velocity and the interest rate for the two alternative sums of financial assets. According to the monetary theory prediction, velocity should increase with the interest rate, for a high opportunity cost will make the holding of money more costly. The simple linear correlation between velocity and interest rate (as given by the overnight rate) was 0.53 for the Divisia aggregate M4 (with monthly data and criterion 2), but only 0.33 for the simple-sum aggregate M4.[24]

Figure 3B
Monetary Multipliers for Divisa M4 and Simple-sum M1 (montly data)



As a final graphic device, in Figures 4A-C we present the terms indicating the money service (V_t) and the investment return (V_t^*) for the simple-sum monetary aggregates M2, M3 and M4, respectively. We see that the investment yield component (given by the gap between the two lines) which is a sort of error-in-the-variable embedded in the use of the simple-sum as an index of monetary wealth, increases in the case of aggregate M2 a great deal over the last few years of the series. The increasing gap poses an upward bias problem when this monetary aggregate is used for policy purposes. Also the fact that the error gap is large this would by itself indicate some inadequacy in the use of the simple-sum aggregate as an indicator of monetary wealth. As the error gap is less pronounced for aggregates M3 e M4, this suggests the error component is smaller for both savings deposits and time deposits than it is for public bonds which enhance the role of such broader aggregates relatively to M2 as tools of monetary policy.

24 These correlation coefficients did not change much when we used for the interest rate instead the rate on the CDB (Certificate of Banking Deposits), which were then 0.532 and 0.388, respectively.

Figure 4A
The Monetary Services (Vt) and the Investiment Income (Vt*) of Simple-sum Aggregate M2 (quarterly data)

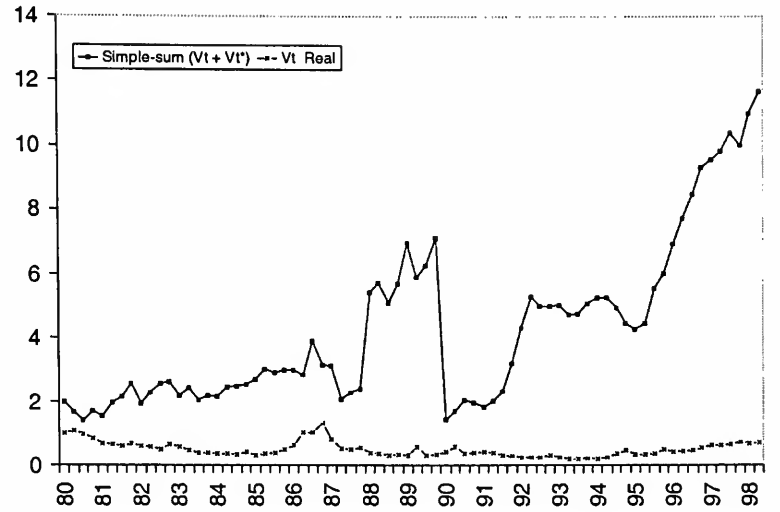


Figure 4B
The Monetary Services (Vt) and the Investiment Income (Vt*) of Simple-sum Aggregate M3 (quarterly data)

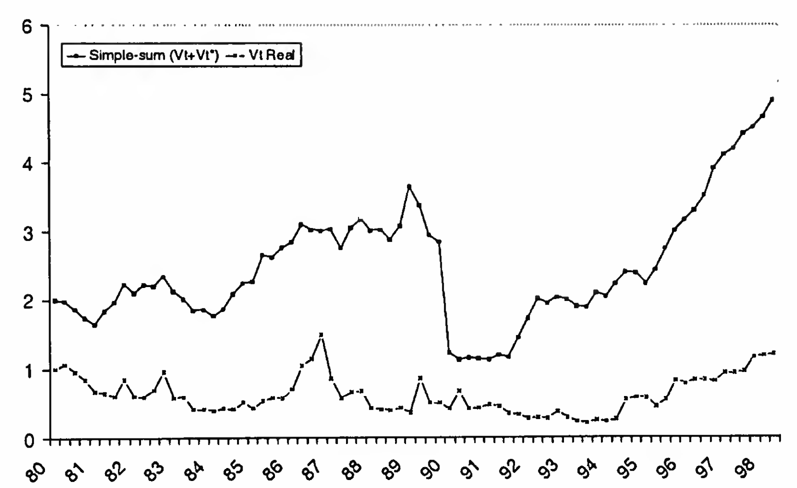
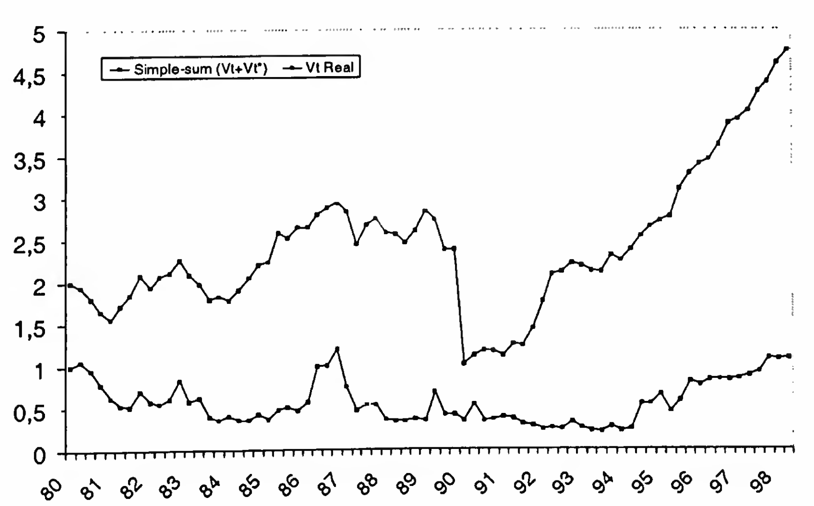


Figure 4C
The Monetary Services (Vt) and the Investiment Income (Vt*) of Simple-sum Aggregate M4 (quarterly data)



Given the differences among the monetary aggregates shown in the figures above, we can now look at a possible superiority of the Divisia aggregate on the basis of its relationship to the inflation rate. With this in mind, Table 1 gives the results of a linear regression between the inflation rate (the dependent variable) and changes in a given monetary aggregate. In the case of the Divisia aggregate, assets were added by both criteria in the choice of the maximum interest rate of the economy as discussed above.

Before analyzing these results, a few words are in order concerning the estimation exercise. Firstly, as the Divisia methodology gives only the rate of change of a given monetary aggregate our regression model is specified in terms of the rate of change of the two variables rather than their levels. Secondly, as the use of nonstationary variables could lead to a spurious regression (unless they cointegrate, i.e. there is a long-run equilibrium relationship among them) such variables were tested for both the presence of a unit root and cointegration. Thirdly, some considerations are undertaken concerning the short-run versus the long-run relationships between the two variables of the model.

Regarding the stationarity test, we applied either the conventional Augmented Dickey-Fuller (ADF) test (with a constant, a trend term and four lags for the first difference variable, and also this same specification but without a trend term) or the Phillips-Perron test (with four truncation). The results (not shown) of these tests indicated that nonstationarity could not be rejected for the two variables of the regression.[25] More precisely, in all cases the two variables were integrated of order one, or I(1); that is, the variables were stationary in their first-differences.[26]

As for the cointegration test, the results (not shown) indicated that in all cases cointegration between the two variables could not be rejected either by the Johansen (1988) or by the Engle and Granger (1987) procedures. Thus, given the fact that a linear combination of the two variables was stable, there is no risk of a spurious regression outcome here.[27] In fact, according

25 One should be reminded here that there are situations in which the power of these tests is very low; as, for instance, in distinguishing between a unit root and a near unit root process. This difficulty has in fact encouraged some researchers to propose alternative procedures in which the unit test hypothesis is altogether eliminated; as is the case, for example, of the Bayesian approach proposed by Sims (1988). On these points see Enders (1995).

26 A plot of the variable series seem to indicate a possible structural change during the Real Plan. We are aware that in the presence of structural breaks both the Dickey-Fuller and Phillips-Perron tests are biased toward the nonrejection of a unit root. In fact, tests for unit root in the presence of structural breaks have been proposed by both Perron (1989), when the time of the breaks is known, and Perron and Vogelsang (1992), when the time of the breaks is not known. We have made no attempt to apply these tests here. In any case, as the regression variables have similar structural changes we hope this will not present any estimation difficulty.

27 Typically, in a spurious regression situation there is a combination of high t-Statistics and low Durbin-Watson, which is not the case for the results in Table 1.

to Stock (1987) the Ordinary Least Square (OLS) estimates are in this case "superconsistent", in the sense that the coefficients of the cointegrating nonstationary variables converge in the regression to their true values much faster than is the case with the coefficients of the corresponding stationary variables. As a note of caution, even though in this case issues concerning the dynamic specification and possible endogeneity of variable x can be ignored asymptotically, in finite samples bias might be a problem. Additionally, as it has been shown in the literature that the distribution of the OLS estimators are in this case non-normal, standard tests of hypothesis are invalidated as well. [28]

As it was emphasized above, the cointegration relationship gives only the long-run equilibrium among the variables. Thus, though the regression model is estimated consistently by OLS, it might face some difficulties in finite samples. This by itself would suggest a search for an alternative model specification. In addition, for some purposes (forecasting, for instance), one is interested in the short-run evolution of the variables, which will require the specification of some dynamic model.[29] With this in mind, suppose the long-run relationship between the two variables (the inflation rate and the changes in the monetary aggregate) is given by

$$y_t = \beta_0 + \beta_1 x_t$$

A particularly simple dynamic model of short-run adjustment is given by[30]

$$y_t = \alpha_o + \gamma_o x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + u_t ,$$

where the residual is white noise having a normal distribution with a zero mean and a constant variance.[31] Here the coefficient of the current value of x indicates the short-run effect of this

28 See on these points, Harris (1995).

29 The discussion, as well as all the equations used below, are based on Harris (1995).

30 In fact, a dynamic model closely resembling this one is obtained from an equation where y depends on current and past values of x, in which the coefficients of the lagged variables decline geometrically (Koyck scheme) or, alternatively, where there is some cost involved in the adjustment of the actual value of variable y to its desired or optimal level. The dynamic model above is easily generalised to allow for more complicated adjustment processes by increasing the lags of the two variables of the model. On these matters, see Harris (1995) and Johnston (1972).

31 Notice that this dynamic model (without a constant) can also be written as:

$$y_t = \beta x_t + \lambda_1 \Delta x_t + \lambda_2 \Delta y_t + v_t$$

where $\beta = (\gamma_0 + \gamma_1)/(1-\alpha); \lambda_1 = -(\gamma_1/1-\alpha); \lambda_2 = -\alpha/(1-\alpha); v_t = u_t/(1-\alpha)$

Thus, the estimation of the long-run model (also without a constant) is equivalent to the estimation of this dynamic model without the terms containing the two first-difference variables, the consequences of which were discussed above in terms of the asymptotic properties (consistence) of the OLS estimators of the long-run relationship versus their finite samples difficulties.

variable on variable y. As for the corresponding long-run effect, if the model were in equilibrium this would be given by the slope of the long-run relationship above.[32]

There are, nevertheless, some potential difficulties in the estimation of the short-run adjustment model as specified above. Firstly, there may be multicollinearity between the current and past values of variable x; therefore, even with a correctly specified model, imprecise parameter estimates might result, and this may be so inspite of a good over all fit of the model. Secondly, as the variables of the model are nonstationary, there is, once again, the risk of a spurious regression outcome.

In an attempt to avoid a spurious regression one might think that a dynamic model reespecified in terms of the first-difference of the nonstationary variables would do it. Unfortunately, this will remove any information concerning the long-run relationship between the two variables, reducing the usefulness of the model for forecasting purposes, for one thing. There is, though, a better procedure in this case. More precisely, the same dynamic model above should be rearranged as follows:

$$\Delta y_t = \gamma_0 \Delta x_t - (1-\alpha_1)(y_{t-1} - \beta_0 - \beta_1 x_{t-1}) + u_t,$$

where $\beta_0 = \alpha_0 / 1 - \alpha_1$ and $\beta_1 = (\gamma_0 + \gamma_1)/(1-\alpha_1)$

This is the so-called error-correction model proposed by Engle and Granger (1987). Like its equivalent short-run adjustment model, it also incorporates both short-run and long-run effects. But, it is more suitable for estimation purposes. The basic reason being as variables y and x cointegrate such an specification would then include only stationary variables, hence reducing the risk of a spurious regression outcome.

As for the estimation of this model, notice, in particular, the coefficient of the variable in parenthesis indicates how the changes in variable y respond to any desequilibrium in the long-run relationship. Although the application of such a model to our monthly and quarterly data indicated that in all cases such a response was positive (as one would expect) and highly statistically significant, unfortunately not much difference in results was found when either the Divisia or the simple-sum monetary aggregates were used.

32 In fact, the long-run effect of x on y based on these two specifications (dynamic model and long-run equilibriun) are related as follows:

$$\beta_1 = (\gamma_0 + \gamma_1)/(1-\alpha_1).$$

With the qualifications above, the regression results of Table 1 are judged here according to two criteria: the regression goodness-of-fit and the closeness to proportionality of the relationship between changes in the monetary aggregate and the rate of inflation. On both accounts, the Divisia aggregate is, in most cases, superior.[33] Also to be noted: the goodness-of-fit always improves in going from the monthly to the quarterly data, which is actually to be expected, for by aggregating the data one almost certainly improves the regression fitting.

Table 1
Dependent Variable: The Inflation Rate (period 1980-1998)

		M2				M3				M4			
		Slope	t	R^2	DW	Slope	t	R^2	DW	Slope	t	R^2	DW
Divisia1	M	0.61	14.0	0.47	1.47	0.69	15.7	0.52	1.69	0.76	17.8	0.58	1.89
	Q	0.98	13.9	0.73	2.47	1.06	16.1	0.78	2.18	1.10	17,0	0.80	2.22
Divisia2	M	0.70	16.1	0.54	1.66	0.75	16.8	0.56	1.83	0.84	19.8	0.63	1.96
	Q	1.03	16.4	0.79	2.56	1.11	19.0	0.83	2.15	1.12	19.5	0.84	2.24
Simple-sum	M	0.63	13.8	0.46	1.48	0.71	14.7	0.49	1.78	0.86	19.4	0.63	1.91
	Q	0.92	9.4	0.55	2.18	1.11	13.2	0.71	1.90	1.11	12.9	0.70	2.09

M and Q stand for monthly and quarterly data, respectively; t is the t-statistic; the intercept was estimated, but is omitted here. Divisia 1 and 2 indicate the two weighting procedures used (as discussed in the text).

An important point in monetary theory concerns the causality between money and prices. Here we applied the Granger causality test, which, as it is known, only gives the temporal precedence of one series over the other. In the testing procedure although two, three and four lags were used in the regression, we report here only the results using the simpler two-lag structure. We found the use of distinct lags did not change the basic conclusions.

Briefly stated, the results (not shown) were as follows: with monthly data, prices Granger-cause money for the Divisia aggregates, while for the simple-sum aggregates causality goes both ways. With quarterly data the causality goes from money to prices for all aggregates, except for the Divisia M3 (criterion 1) where neither money causes price nor prices cause money.[34] These results are actually in line with the spirit of the monetary theory that predicts a long-run

33 In view of the same zero interest rate considered here for either currency or demand deposits, then the simple-sum M1 and the Divisia M1 would coincide. Thus, such an aggregate is not included in Table 1. In any case, just for a comparison the use of M1 in the exercise of Table 1 produced, with monthly data, a slope coefficient of 0.98 (t-statistics of 12.2) with R^2 =0.41, while the corresponding figures for the quarterly data were a slope of 1.07 (t-statistic of 16,4) with R^2 = 0.79.

relationship that goes from money to prices. This is more clearly established with quarterly rather than monthly data. Perhaps the lag structure used here was not long enough to capture such a basic monetarist dogma when monthly data were used for the Divisia aggregates. In conclusion, although the results with quarterly data seem to indicate that the causality from money to prices is more clearly established in the case of the simple-sum aggregates, in the overall picture it seems there is really no clear-cut winner on the matter.

4 Final remarks

We have shown that in general the Divisia monetary aggregates produced better results here than the simple-sum of assets. The monetary aggregates, which were considered in the analysis, are those traditionally defined by the Central Bank. In the Divisia monetary aggregation methodology it is assumed that the group of monetary assets is weakly separable from other goods and services in the individual's utility function. This suggests that a distinct monetary aggregate from those in use may very well be preferable. As our interest here was merely compare the results of the Divisia weights with those of the simple-sum monetary aggregates as the latter are currently defined by the Central Bank, we have not attempted to test for separability in this study. The performance of a Divisia aggregate which meets the separability test will probably be better than the one indicated in this study. Most certainly, such an aggregate would involve other combinations of assets than the ones actually used by the traditional aggregates. This matter is sufficiently complex to deserve a separate study.

Among other points deserving further investigation, one should probably include the question of the money demand stability when alternative schemes of adding assets are used, as well as the sensibility of the Divisia results to either changes in the choice of the maximum interest rate in the economy, or to the use of distinct interest rates for currency and demand deposits. As these are all very relevant subjects, and in view of the extensive nature of such topics, they obviously deserve to be investigated in separate studies.

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